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MOVING FORCE IDENTIFICATION STUDIES, II: COMPARATIVE STUDIES

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The theoretical background for four moving force identification methods has been presented in a companion paper. This part of the paper is an extension of the work to study the applicability of the four moving force identification methods. The parameters under study include the speed of vehicles, sampling frequency, axle-spacing-to-span ratio, and sensitivity towards noise. For the time domain method and frequency–time domain method, recommendation of the number of strain gauges is included. A comparison of the accuracy of the force identification using the four methods is given. Both illustrative and experimental results show that each method has its merits and limitations. It is found that the TDM gives the best results, but it is time consuming. Recommendations are given for each method to improve their performance and range of application. Further investigation would be to merge the four identification methods into a moving force identification system (MFIS).

1. INTRODUCTION

Force identification is an inverse problem in structural mechanics. In the field of bridge engineering, a number of force identification techniques have been developed [1–6]. However, these techniques measure only static axle loads. Hoshiya and Maruyama [7] suggested an advanced method for identification of a moving load running on beams. O'Connor and Chan [8] suggested another advanced force identification method—the interpretative method I (IMI), which is able to measure both dynamic and static axle forces of multi-axle systems. A number of references [9–11] show that acceptable results identified from responses when noise is added can be obtained using a system identification method. This prompts investigation into the use of system identification method for moving force identification. Four moving force identification methods have been developed by the authors. Two methods, namely, the time domain method (TDM) [12], and the frequency-time domain method (FTDM) [13] have been developed based on a system identification method. The fourth method, which is called the interpretative method II (IMII) [14], is similar to IMI. The theoretical background and a preliminary study of the four moving force identification methods have been introduced in the companion paper [15]. The TDM has also been applied successfully to a field study [16]. All these methods can identify moving forces with acceptable accuracy. However, each method has its own

TABLE 1

A summary of the initial values of parameters

Span length (m)	Flexural stiffness ($\times 10^{11}$ N m^2)	Unit mass (kg/m)	Axle spacing (m)	Axle (1) weight (N)	Axle (2) weight (N)	Speed (m/s)	Sampling frequency (Hz)
40	1·27914	12 000	8	58 800	137 200	40	200

merits, limitations and disadvantages. This paper aims to critically investigate all these methods through illustrative examples and experiments in laboratory based on a common scheme for the comparative study.

2. PRELIMINARY CONSIDERATION

2.1. ACCEPTABLE CRITERION

On comparing identified forces, the accuracy is quantitatively defined as a percentage error E_{error}, sometimes also called relative percentage error (RPE). It is the absolute value of the sum of the differences between the discrete identified (ε_{ident}) and true (ε_{true}) axle loads over the total of the true axle loads over a period of time, as

$$E_{error} = \frac{\sum|\varepsilon_{true} - \varepsilon_{ident}|}{\sum|\varepsilon_{true}|} * 100\%. \tag{1}$$

The maximum acceptable percentage error adopted in this study is 10%.

2.2. INITIAL PARAMETERS

A set of parameters is chosen as the initial bridge and vehicle parameters for the simulation of dynamic responses, i.e., bending moments and accelerations, as input data for force identification. IMI and IMII were implemented using only bending moments as input data and TDM and FTDM were implemented using bending moments and accelerations as input data. The set of initial parameters including bridge and vehicle properties is summarized in Table 1.

In order to analyze a wide range of case studies, an initial axle spacing of 8 m, a span length of 40 m, and an initial speed of 40 m/s (144 km/h) were chosen. The first few vibration modes are important in the vibration analysis of dynamic systems. For a short-span highway bridge, the natural frequency of the third vertical flexural vibration mode is about 40 Hz. Therefore, the frequency range between 0 and 40 Hz is the bandwidth of interest, and will be referred to as the analysis frequency bandwidth (AFB). The initial sampling frequency of 200 Hz was chosen because both TDM and FTDM require the sampling frequency to be at least 5 times that of the AFB. In addition to the above-specified parameters, in the simulation of the four identification methods, the number of computation steps has to be specified. This value specifies the total number of time steps of discrete dynamic responses to be generated. In this study, this number was initially chosen as 512 steps which is greater than the total number of steps ($240 = \{40\text{ m} + 8\text{ m}\} * 200\text{ Hz}/40\text{ m/s}$) taken by the vehicle moving from one end to the other.

Table 2

A summary of sensor arrangements for TDM and FTDM

Location	Case											
	i	ii	iii	iv	v	vi	vii	viii	ix	x	xi	xii
1/4a			o	o			o	o	o	o	o	o
1/2a			o	o	o	o	o		o			o
3/4a				o								
1/4m	o	o				o	o	o	o		o	
1/2m	o	o			o	o	o			o	o	o
3/4m		o										

Note: o—sensor location, a—using accelerometers, m—using strain gauge; 1/4, 1/2, 3/4—quarter, mid, three-quarter span respectively.

2.3. SENSOR ARRANGEMENT

In the IMI and IMII studies, all sensors are equally distributed along the beam. In the TDM and FTDM studies, 12 sensor arrangement cases were initially implemented in one computer program using MATLAB. Herein the measurements using accelerometers and strain gauges are referred to as "a" and "m" respectively. In Table 2, 1/4a, 1/2a, or 3/4a indicates having an accelerometer at quarter-span, mid-span, or three-quarter span respectively. Similarly, 1/4m, 1/2m, or 3/4m means having a strain gauge at quarter-span, mid-span, or three-quarter-span respectively. One of the 12 sensor arrangement cases will be selected as the best sensor arrangement for two-axle force identification following the comparative study scheme described below.

3. SCHEME OF STUDY

The comparative study scheme includes the following: a two-axle constant force identification study, a multi-axle identification study, a time-varying force identification study, and a measurement error study. The comparative study plan is shown in detail in Figure 1.

3.1. TWO-AXLE CONSTANT FORCE IDENTIFICATION STUDY

As the speed of the vehicle, the sampling frequency, the axle-spacing-to-span ratio (ASSR), and the level of noise are fundamental input data for the force identification, it is interesting to examine their effects on the accuracy of identified forces. One parameter is studied at a time. The examination sequence is the sampling frequency, the speed of the vehicle, the axle spacing, and the noise level.

3.2. MULTI-AXLE IDENTIFICATION STUDY

The purpose of the multi-axle identification study is to investigate the minimum required number of sensors for multi-axle force identification. O'Connor and Chan [8] and Chan *et al.* [14] have proposed formulae for the determination of the required number of strain

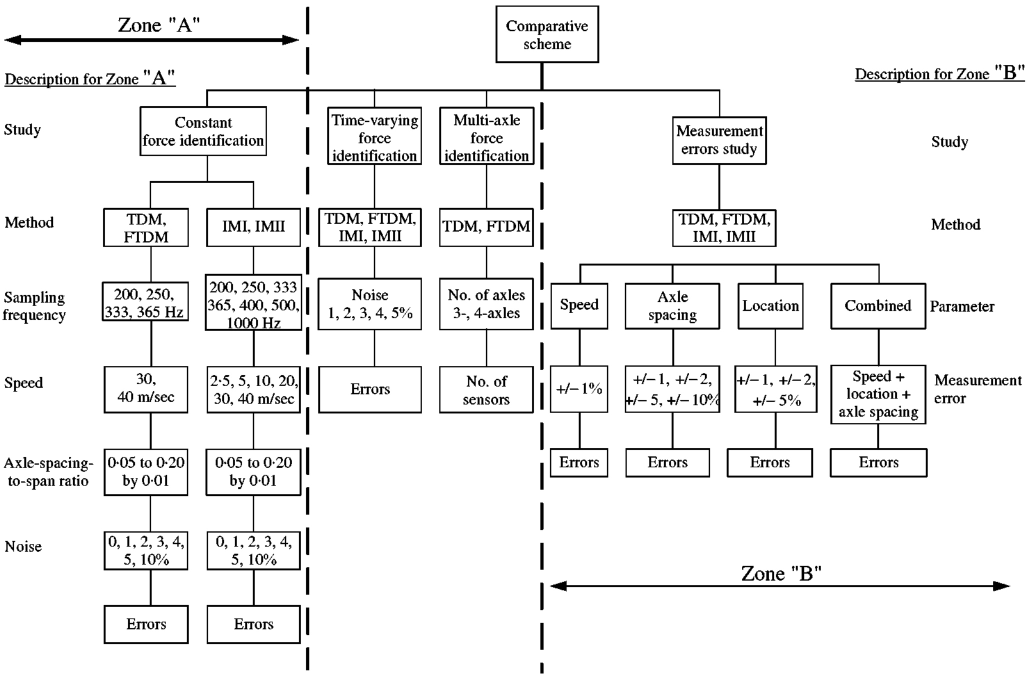
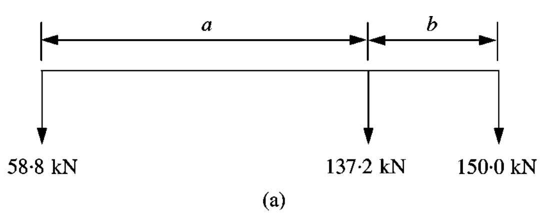
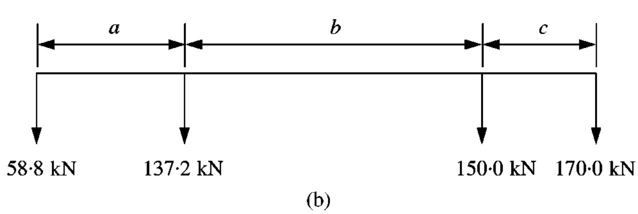


Figure 1. Summary of study plan.



Case	a	b
1	3·0 m	2·5 m
2	3·4 m	1·6 m
3	3·5 m	1·3 m



Case	a	b	c
1	2·0 m	6·0 m	1·5 m
2	2·5 m	4·0 m	1·5 m
3	3·0 m	4·0 m	1·3 m

Figure 2. The three- and four-axle vehicle models studied: (a) a three-axle vehicle model; (b) a four-axle vehicle model.

gauges for IMI and IMII respectively. Therefore, only TDM and FTDM are to be investigated in this study. One and two additional axle forces are to be added to form 3- and 4-axle vehicle models. Figure 2 shows a typical configuration of the 3- and 4-axle vehicle models. In order to obtain an empirical formula for the determination of the recommended number of sensors, a number of cases have to be studied. These cases include using different span lengths (50, 40, 30, 20 m), speeds of vehicles (40, 30 m/s), ASSRs (0·2 down to 0·1 $\times$ 0·01), and sampling frequencies (200, 250, 333 Hz).

3.3. TIME-VARYING FORCE IDENTIFICATION STUDY

This study verified the four methods for application to the identification of time-varying forces. The time-varying forces are similar to constant forces except that they oscillate about their means with an amplitude of 10% of their means. It is assumed that the results of the time-varying force identification are similar to the results of the constant force identification and only five levels of noise were studied. The initial parameters for the bridge and vehicle models were used.

3.4. MEASUREMENT ERROR STUDY

In practice, bridge and vehicle measured parameters may include errors. This study investigates the accuracy of identified forces with three parameters containing errors, namely the speed of vehicles, the axle spacing of vehicles and locations of measuring sensors.

4. RESULTS OF ILLUSTRATIVE STUDY

4.1. TWO-AXLE IDENTIFICATION STUDY

4.1.1. *Effect of sampling frequencies*

On testing different sampling frequencies, no errors are obtained using TDM and FTDM. As there is a computer memory problem with the computation of the inverse of a large matrix, the maximum sampling frequency is limited to 365 Hz. For the IMI and IMII cases, zero errors could not be obtained but the results could still be considered very accurate (maximum error 2·5%). The small percentage error is due to the use of different differentiation schemes in the forward and inverse processes. In the simulation, dynamic responses of IMI and IMII are obtained using Newmark's method and the Runge–Kutta method respectively. However, the velocities and accelerations are obtained using the central difference method in the force identification. The results show that the errors are similar in each case. This implies that all four methods are therefore independent of the sampling frequency.

4.1.2. *Effect of speed of vehicles*

It is found from Table 3 that the effect caused by the decrement of the speed is similar to that of an increase in sampling frequency. For IMI and IMII, the speeds studied varied from 40 down to 2·5 m/s with the error found being less than 1·3%. For TDM and FTDM, no error is induced at the speed of 40 m/s. However, an error is induced at the second axle at the speed of 30 m/s. At the speed of 30 m/s, an example using a larger value of the number of computation steps (630 points) was studied. The results show that no error is identified. This implies that the number of computation steps has to be larger than the total number of time steps by the time the last axle force moves off the beam. For subsequent studies, 630 computation steps were adopted.

4.1.3. *Effect of ASSRs*

The ASSRs varying from 0·2 down to 0·05 were tested except in IMI where an ASSR down to 0·1 was tested. Table 4 lists some of the percentage errors with various ASSRs. For IMI, results show that a larger ASSR produces a higher percentage error between the true

TABLE 3

A summary of the percentage errors of the identified forces with various speeds (( )-*using* 630 *steps*)

Speed (m/s)	Percentage error (%)							
	IMI		IMII		TDM		FTDM	
	SF = 500 Hz		*SF* = 500 Hz		*SF* = 365 Hz		*SF* = 365 Hz	
	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2
40	0·782	0·379	1·149	1·030	0·0	0·0	0·0	0·0
30	1·299	0·418	1·055	0·908	0·0	38·1 (0·0)	0·0	81·9 (0·0)
20	0·614	0·281	0·868	0·686	N/A	N/A	N/A	N/A
10	0·896	0·418	0·723	0·500	N/A	N/A	N/A	N/A
5	0·444	0·205	0·650	0·392	N/A	N/A	N/A	N/A
2·5	0·391	0·209	0·603	0·327	N/A	N/A	N/A	N/A

TABLE 4

A summary of the percentage errors of the identified forces with various ASSRs

Ratio	Percentage error (%)							
	IMI		IMII		TDM		FTDM	
	Span = 40 m; *SF* = 500 Hz; Speed = 5 m/s		Span = 40 m; *SF* = 500 Hz; Speed = 5 m/s		Span = 40 m; *SF* = 365 Hz; Speed = 30 m/s		Span = 40 m; *SF* = 365 Hz; Speed = 30 m/s	
	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2
0·20	0·444	0·205	0·65	0·392	0·0	0·0	0·0	0·0
0·15	0·681	0·252	0·689	0·345	0·0	0·0	0·0	0·0
0·12	231·3	45·0	0·702	0·311	0·0	0·0	0·0	0·0
0·10	342·1	105·5	0·723	0·306	0·0	0·0	0·0	0·0
0·09	N/A	N/A	0·804	0·278	0·0	0·0	0·0	0·0
0·05	N/A	N/A	1·358	0·539	0·0	0·0	0·0	0·0

and identified moving forces. Especially, errors larger than 10% are obtained at and below the ratio of 0·12.

For IMII, the errors also increase with a smaller ASSR. However, it is shown that acceptable results can be obtained for values as small as 0·05. For TDM and FTDM, results show that there is no adverse effect by reducing the ASSR. The corresponding percentage errors are found to be zero all the time. This implies that the accuracy of identified forces using both methods is not dependent on the ASSR.

4.1.4. *Noise effect*

Both expected and unexpected signals will be measured using a data acquisition system during measurement. Unexpected signals called measurement noise are always induced from electrical devices of the acquisition system. It is therefore interesting to study the effect

TABLE 5

A summary of the percentage errors of the filtered identified forces

Noise (%)	Percentage error (%) IMI: Span = 40 m; SF = 500 Hz; Speed = 5 m/s; Ratio = 0·1		IMII: Span = 40 m; SF = 500 Hz; Speed = 5 m/s; Ratio = 0·1		TDM: Span = 40 m; SF = 365 Hz; Speed = 30 m/s; Ratio = 0·1		FTDM: Span = 40 m; SF = 365 Hz; Speed = 30 m/s; Ratio = 0·1	
	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2
1	—	72·7	2·23	1·30	1·88	1·74	3·92	2·11
2	—	72·1	4·18	2·32	3·24	2·91	7·27	3·65
3	—	78·8	6·19	3·38	4·64	4·09	10·63	5·19
4	—	—	8·21	4·44	6·09	5·26	13·98	6·74
5	—	—	10·3	5·51	7·58	6·44	17·34	8·28
10	—	—	20·5	10·8	15·15	12·31	34·16	16·01

— means errors greater than 100.

of noise on the accuracy of identified forces. All four methods were tested with seven levels of noise. Obviously, the best result is obtained from a noise level of 0%. In this study, the results are filtered using a low-pass filter.

In the unfiltered situation, IMI will produce large percentage errors even with a noise level of 1%. It is observed that two problems cause such high percentage errors. One is that IMI is not good for identifying moving forces with a small ASSR, and the other comes from numerical differentiation. As a large percentage error is already induced due to using a small ASSR, there is an inherently large percentage error in the identified axle forces and the error will be amplified using numerical differentiation. When the ASSR increases to 0·15 then the errors corresponding to noise of 1% are significantly reduced.

For cases of no noise, it has been shown that IMII can be used to identify a value of ASSR as small as 0·05. Results show that the percentage error is proportional to the noise level. For TDM and FTDM, the input responses are bending moments and accelerations. However, the accelerations are not derived from the input bending moments. This differs from IMI and IMII. As numerical differentiation is not necessary, no error amplification is caused by using a numerical differentiation method. The first three best combinations of TDM and FTDM are cases iv, vii, and xii (refer to Table 2 for sensor arrangement). The order of sensitivity towards noise for all the four methods is found to be (1) IMI, (2) IMII, (3) FTDM, and (4) TDM.

Percentage errors of filtered results using the four methods are tabulated in Table 5 in which the percentage errors of IMI are still very high after filtering. This is because the ASSR is too small to identify. If the ASSR was increased to 0·15, the errors for the cases with 1% noise after filtering would be reduced to 9·93 and 3·98%. Both filtered identified forces are acceptable.

4.2. MULTI-AXLE IDENTIFICATION STUDY

4.2.1. *Recommended number of sensors*

For the three-axle force identification, results show that all combinations can correctly identify all three axle forces except those combinations with only two sensors, i.e., cases i, iii,

Table 6

A summary of sensor arrangement for three-axle force identification

Location	Case									
	i	ii	iii	iv	v	vi	vii	viii	ix	x
1/5a	o	o			o		o		o	
2/5a	o	o	o	o		o	o	o	o	
3/5a	o	o	o	o			o	o	o	
4/5a									o	
1/5m	o		o		o	o		o		o
2/5m		o	o	o	o	o	o	o		o
3/5m				o	o	o	o	o		o
4/5m										o

Note: o—sensor location, a—using accelerometers, m—using strain gauge; 1/5, 2/5, 3/5, 4/5—one-, two-, three- and four-fifth span respectively.

Table 7

A summary of sensor arrangement for four-axle force identification

Location	Case										
	i	ii	iii	iv	v	vi	vii	viii	ix	x	xi
1/6a	o		o				o				
2/6a	o		o		o	o	o	o	o	o	o
3/6a	o		o	o	o	o	o	o	o		o
4/6a	o		o		o				o	o	o
5/6a	o										
1/6m		o		o				o			
2/6m		o		o	o	o	o	o	o	o	o
3/6m		o	o	o	o	o	o	o		o	o
4/6m		o		o		o			o	o	o
5/6m		o									

Note: o—sensor location, a—using accelerometers, m—using strain gauge; 1/6, 2/6, 3/6, 4/6, 5/6—one-sixth, one-third, mid, two-third and five-sixth span respectively.

v, viii, and x in Table 2. In these cases the number of sensors is less than the number of axles and therefore the number of equations is less than the number of unknowns. A new set of sensor arrangements for 3-axle force identification is defined and tabulated in Table 6.

As shown above, the sampling frequency and ASSR have no adverse effect on force identification, but it is interesting to study what combination is the best for identifying axle forces when noise is added to the input data. Results show that the best sensor arrangement is case ix in Table 6 with all sensors being accelerometers. This result is similar to the two-axle force identification study. For four-axle force identification, again, a new set of sensor arrangements for 4-axle force identification is defined and tabulated in Table 7.

Results show that the best combination is case i in Table 7 with all sensors being accelerometers. Based on the two-, three-, and four-axle force identification studies, it is concluded that the recommended number of sensors should be equal to or greater than the number of axles plus one. The best combination of the recommended number of sensors is

TABLE 8

A summary of the percentage errors of time-varying force identification

Noise %	Span length = 40 m, speed = 40 m/s, sampling frequency = 200 Hz							
	Percentage error (%)							
	IMI		IMII		TDM-case iv		FTDM-case iv	
	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2
0	1·3	0·791	1·6	1·9	0·0	0·0	0·0	0·0
1	—	—	88·7	47·2	0·908	0·93	1·9	1·4
2	—	—	—	93·7	1·8	1·9	3·8	2·8
3	—	—	—	—	2·7	2·8	5·7	4·2
4	—	—	—	—	3·6	3·7	7·7	5·6
5	—	—	—	—	4·5	4·7	9·6	7·0

— means errors greater than 100.

for all sensors to be accelerometers. The percentage errors are all zero for multi-axle force identification when no noise is added to the input data.

4.2.2. *Time-varying force identification study*

The four methods were tested with a set of time-varying sinusoidal forces. Results show that the four methods can also be used to identify the set of time-varying forces. When no noise is added, the identified forces for all methods are identical to the true values except for IMII in which minor errors are observed. For TDM and FTDM, the best sensor arrangement for time-varying force identification is case iv. This is consistent with the constant force identification study. A summary of the percentage errors between the true and identified forces is tabulated in Table 8. Figures 3 and 4 show the unfiltered and filtered time-varying axle forces respectively.

4.3. MEASUREMENT ERRORS STUDY

4.3.1. *Speed error*

In this part of the study, two types of speed errors were considered. One is due to the variation of speeds (Type-I error), the other is due to the measurement error (Type-II error). The identified forces the Type-I error cases are obtained using an equivalent model. The identified forces for Type-II error cases are obtained using the true model taking measurement errors into consideration. Table 9 shows that only TDM gives acceptable identified forces accuracy.

4.3.2. *Location error*

For "ahead" and "behind" (see Table 10) cases, the percentage errors are almost proportional to the percentage difference in the first column in Table 10 for the four identification methods. The percentage errors for "ahead" cases are similar to "behind" cases for the four identification methods. For both TDM and FTDM, the percentage errors "alternative" cases are also similar to both the "ahead" and "behind" cases. For IMI and

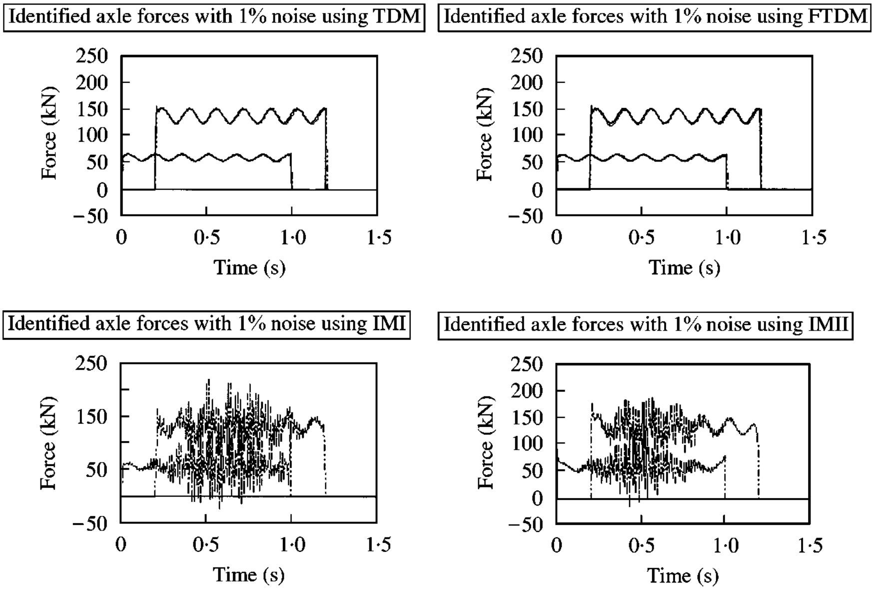


Figure 3. Examples of the identification of time-varying axle forces with 1% noise: ——, true axle force; – – – –, identified axle force.

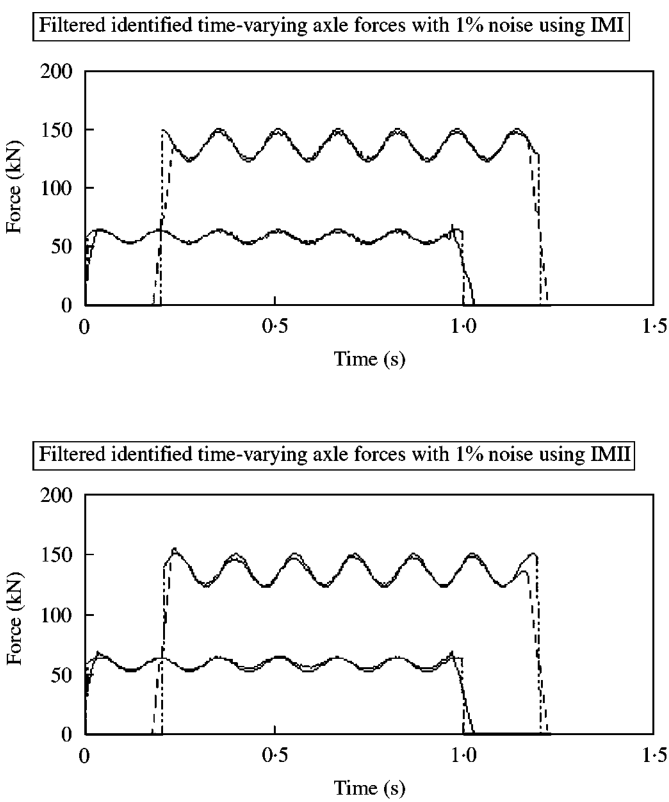


Figure 4. Examples of the identification of time-varying forces with 1% noise: ——, true axle force; – – – – –, identified axle force.

Table 9

Percentage errors of identification using speeds with errors ("*" *means Type*-I *error*; "**" *means Type*-II *error*; — *means errors greater than* 100)

		Span = 40 m, Speed = 40 m/s, Ratio = 0·2							
		IMI		IMII		TDM		FTDM	
Change of speed	Type	Axle 1 (%)	Axle 2 (%)	Axle 1 (%)	Axle 2 (%)	Axle 1 (%)	Axle 2 (%)	Axle 1 (%)	Axle 2 (%)
1%*	accel	58·86	30·25	—	35·12	1·65	0·74	—	—
1%*	decel	65·57	32·25	—	35·20	10·36	11·19	—	—
1%*	alter	94·76	31·17	—	35·40	53·41	26·45	—	—
1%**	constant	—	45·50	6·08	16·67	5·96	4·13	—	—
− 1%**	constant	—	31·98	22·06	4·29	20·34	18·08	—	—

Note: "accel" means the speed is uniformly accelerated, e.g., 40 → 40·4 m/s; "decel" means the speed is uniformly decelerated, e.g., 40 → 39·6 m/s; "alter" means the speed is uniformly accelerated and decelerated, e.g., 40 → 40·4 → 40 m/s; "constant" means the speed is kept constant, e.g., 40 → 40 m/s.

Table 10

Percentage errors of identification using locations with errors

		Span = 40 m	Speed = 40 m/s	Ratio = 0·2					
		IMI		IMII		TDM		FTDM	
Difference (%)	Case	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2
1	ahead	48·4	19·6	10·2	4·5	1·6	0·7	7·6	1·8
2	ahead	97·8	39·2	18·9	9·0	3·1	1·4	15·0	3·5
5	ahead	—	58·7	26·9	13·5	4·7	2·1	22·1	5·2
1	behind	47·1	20·1	9·0	5·7	1·6	0·7	7·8	1·8
2	behind	92·6	40·1	17·2	10·4	3·2	1·4	15·9	3·7
5	behind	—	60·3	25·1	15·1	4·8	2·1	24·1	5·6
1	alternative	—	—	38·5	21·8	1·7	0·7	7·3	1·7
2	alternative	—	—	63·1	42·1	3·4	1·4	16·1	3·7
5	alternative	—	—	75·4	57·5	5·2	2·1	24·5	5·7
1	gauge 1	15·8	26·7	9·0	5·7	0·8	0·7	3·9	1·1
1	gauge 2	24·2	23·8	12·5	5·9	0·4	0·2	0·7	0·2
1	gauge 3	21·9	15·2	16·6	8·2	3·5	0·2	10·9	2·8
1	gauge 4	20·1	9·0	21·1	10·3	N/A	N/A	N/A	N/A
1	gauge 5	39·4	12·9	16·7	8·4	N/A	N/A	N/A	N/A
1	gauge 6	65·1	16·7	16·5	6·4	N/A	N/A	N/A	N/A
1	gauge 7	79·8	15·2	12·9	6·0	N/A	N/A	N/A	N/A

Note: Case ahead—true location of the sensor is behind the location studied; Case behind—true location of the sensor is in front of the location studied; Case alternative—one sensor is ahead; one sensor is behind; and so forth. — means errors greater than 100.

IMII, the percentage errors for the "alternative" cases are worse than for the other two cases.

The worst result is the "alternative" case even when the difference is 1% for both IMI and IMII. For an individual sensor with errors in the locations of sensors for IMII, the results in Table 10 shows that if sensors are closer to mid-span then the reduction of accuracy will be

TABLE 11

A summary of percentage errors of using axle spacings with errors

	Span = 40 m		Speed = 40 m/s		Ratio = 0·2			
	IMI		IMII		TDM case iv		FTDM case iv	
Difference	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2
+ 1%	0·7	1·7	2·9	3·5	0·0	0·0	0·0	0·0
+ 2%	0·8	4·3	4·0	6·8	8·5	9·2	34·4	20·8
+ 5%	1·0	7·0	7·4	8·6	16·9	13·0	73·5	25·6
+ 10%	1·2	14·7	14·1	15·5	32·7	17·9	—	84·0
− 1%	0·7	1·7	2·4	1·1	0·0	0·0	0·0	0·0
− 2%	0·8	4·4	3·1	2·4	7·7	9·0	68·5	85·6
− 5%	1·1	6·9	6·2	5·9	17·0	14·0	—	—
− 10%	1·8	14·7	12·8	12·5	34·9	21·6	—	—

— means errors greater than 100.

more significant. For both IMI and FTDM, if sensors are closer to a support then the reduction of accuracy will be more significant.

4.3.3. *Axle spacing error*

For the IMI and IMII, the results in Table 11 show that the percentage errors for a smaller axle spacing are similar to those for a larger axle spacing. For both TDM and FTDM, results show that the percentage errors for larger axle spacings are usually smaller than for smaller axle spacings. Comparing the percentage errors, the result shows that IMI is the method least sensitive to axle spacing errors for identifying axle forces.

In using IMI and IMII, as the distance from the first axle away to a reference point is independent of the axle spacing, correct axle forces are always obtained except for the range between the true and incorrect entry points. The location of the second axle, however, does depend on the axle spacing and incorrect axle forces are always obtained. Therefore, larger errors are obtained at the second axle rather than at the first axle.

In using TDM and FTDM, a small difference in axle spacing does not affect the accuracy of identified forces, but a large error is induced when the error in the axle spacing is larger, especially with the FTDM. This is because only a small part of the matrix containing the influence coefficients is incorrect and therefore an acceptable result can be obtained. However, that part of the matrix containing the influence coefficients becomes larger when the error becomes larger.

4.3.4. *Combined error*

Table 12 shows the percentage errors of the combined effects of measurement errors in axle spacing, speed of the vehicle (Type-I error), and the location of the sensors. The results of using the speed of vehicle with Type-II errors are similar. Obviously, Table 12 shows that TDM performs the best and FTDM is the method most sensitive to measurement errors. This result agrees with the results in the previous paragraphs. The speed errors produce the dominant adverse effects. Even though some combinations of the error types are beneficial, that beneficial effect is not sufficient to compensate the adverse effects.

TABLE 12

A summary of percentage errors of the combined study

Span = 40 mm Speed = 40 m/sec Ratio = 0·2 Type-I error										
Error			IMI (%)		IMII (%)		TDM (%) case iv		FTDM (%) case iv	
Speed	Location	Axle Sp.	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2	Axle 1	Axle 2
+ 1%	+ 1%*	+ 1%	—	65·72	—	52·50	2·39	1·03	—	—
+ 1%	+ 1%*	− 1%	59·94	99·81	—	36·41	2·25	1·02	—	—
+ 1%	− 1%**	+ 1%	—	30·79	91·37	34·99	2·25	1·02	—	—
+ 1%	− 1%**	− 1%	58·09	32·17	—	35·46	2·39	1·03	—	—
− 1%	+ 1%*	+ 1%	—	30·47	—	35·11	16·26	15·13	—	—
− 1%	+ 1%*	− 1%	67·43	32·21	—	35·04	10·39	11·15	—	—
− 1%	− 1%**	+ 1%	—	68·92	89·29	66·93	10·71	11·28	—	—
− 1%	− 1%**	− 1%	64·10	57·62	—	50·75	10·36	11·15	—	—

— means errors larger than 100; * means ahead; ** means behind.

4.4. DISCUSSIONS

The various factors affecting the accuracy of identified forces have been given in the previous sections. The following section discusses what method should be used in an actual test under various conditions.

(1) IMI and IMII have been developed using only bending moments as input data. Bending moments are always indirectly measured using strain gauges. The other two methods use accelerations and bending moments as input data. A measurement using strain gauges is usually cheaper than using accelerometers. However, installation of strain gauges always takes longer than the installation of accelerometers. A calibration process is also required to convert the measured strain data to bending moments. For a measurement using accelerometers, no such process is required. As IMI and IMII cannot use measured data from accelerometers, TDM and FTDM are more convenient.

(2) The measured locations of sensors may include errors. One of the studies shows that TDM is the least sensitive method if sensor location measurement errors occur. FTDM becomes second and IMI the last.

(3) Speeds are possibly the most difficult to be correctly measured as the speeds of vehicles vary significantly with traffic conditions and individual drivers. The study given above shows that only TDM gives acceptable results when errors exist in the vehicle speed estimation.

(4) Errors in speeds affect the accuracy of identified forces and will also affect indirectly the calculation of axle spacings. If errors in axle spacings are involved in force identification, results show that IMI is the best method for obtaining a higher accuracy of identified forces. FTDM is the worst method.

(5) As measurement errors may exist in more than one parameter, a study of the effect of measurement errors in combination shows that TDM and FTDM are, respectively, the best and worst methods.

(6) Table 7 shows that IMI is not applicable to a small ASSR. IMII is applicable for identification if ASSR is as small as 0·05. TDM and FTDM are good when the

number of computation steps is equal to or greater than the actual number of time steps of the moving force passing over the beam, otherwise an error is induced. In other words, the number of equations for computation should be equal to or greater than the number of unknown forces.

(7) Results show that IMI and IMII require about 1 min using Pentium III to complete a case study, but TDM and FTDM are very time consuming. TDM and FTDM require about an hour to complete a case study. Therefore, TDM and FTDM are not good for real-time force identification.

(8) Besides being time consuming, both TDM and FTDM require a minimum of 16 MB RAM and 150 MB computer memory for computation.

The speed of vehicles is the critical factor in deciding which method should be adopted because speeds are difficult to measure accurately and errors affect the accuracy of the identified forces. It is concluded from these results that TDM is the best method for force identification except for real-time force identification. The second, the third and the last method are IMI, IMII and FTDM respectively.

4.5. RECOMMENDATIONS

(1) Filtered identified forces should be used when input responses include noise.

(2) All methods except FTDM have been implemented to identify axle forces with time-varying speeds. It is recommended to measure the speeds of vehicles at different locations on a bridge rather than to use one equivalent speed in order to increase the accuracy of identified forces.

(3) A preliminary study showed that TDM and FTDM are not suitable for real-time force identification; however, IMI and IMII are suitable. TDM and FTDM, nevertheless, are good at identification with a small ASSR producing a high accuracy of identified forces. As the power of personal computers continues to improve, it is expected that the computation time will greatly reduce in the near future.

(4) Monitoring is important when field measurements are carried out. In order to facilitate monitoring, the governing equations of TDM (refer to reference [15] for notations) should be modified as

$$\begin{Bmatrix} m(2) \\ m(3) \\ \vdots \\ m(N) \end{Bmatrix} = \sum_{n-1}^{x} C_{xn} \underbrace{\begin{bmatrix} E_n^1 S_1(1) & 0 & \cdots & 0 \\ E_n^2 S_2(2) & E_n^1 S_1(1) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ E_n^{N-1} S_1(N-1) & E_n^{N-2} S_1(N-2) & \cdots & E_n^{N-N_B+1} S_n(N-N_B+1) \end{bmatrix}}_{Matrix\ \mathrm{I}}$$

$$\times \underbrace{\begin{bmatrix} S_2(1) & 0 & \cdots & 0 \\ 0 & S_2(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_2(N_B-1) \end{bmatrix}}_{Matrix\ \mathrm{II}} \begin{Bmatrix} f(1) \\ f(2) \\ \vdots \\ f(N_B-1) \end{Bmatrix}. \tag{2}$$

As Matrix I is independent of time, its generation and subsequent inversion can be done off-site once bridge parameters are known. Hence, the time of real-time computation would further be significantly reduced.

(5) Both strain gauges and accelerometers are commonly used for measurements. The strain gauges and accelerometers are used at a node to acquire bending moments and accelerations respectively. In order to broaden the application and to increase the accuracy of IMI and IMII, it is recommended to improve IMI and IMII so that not only bending moment responses, but also acceleration responses can be used as input data.

5. EXPERIMENTS IN LABORATORY

After the effect of various parameters on the identification accuracy of each method had been evaluated through illustrative examples, a series of experiments were conducted in laboratory for further robustness assessment of all the four identification methods. Both the model car and model bridge deck were constructed in the laboratory as shown in Figure 5. Here, the model car had two axles at a spacing of 0·55 m and was mounted on four rubber wheels. The static mass of the whole vehicle was 12·1 kg in which the mass of rear wheel was 3·825 kg. The model bridge deck consisted of a main beam, a leading beam and a trailing beam. The main beam with a span of 3·678 m length and 101 mm × 25 mm uniform cross-section, was simply supported. It was made from a solid rectangular mild steel bar with a density of 7335 kg/m^3 and a flexural stiffness $EI = 29{\cdot}97$ kN/m^2. A U-shape aluminum track was glued to the upper surface of the main beam as a guide way for the model car, which was pulled along by a string wound around the drive wheel of an electric motor. The speed of the motor could be adjusted in order to get a specific car speed. Seven photoelectric sensors were mounted beside the beams to measure and check the uniformity of moving speed of the model car.

Seven equally spaced strain gauges and three equally spaced accelerometers were mounted on the lower surface of the main beam to measure the bridge response due to the moving across car. A system calibration of the strain gauges was carried out before the actual testing program by adding masses at the middle of the main beam. A 14-channel tape recorder was employed to record the response signals. The software Global Lab from the Data Translation was used for data acquisition and analysis in the laboratory test. Before exporting the measured data in ASCII format for identification, the Bessel IIR digital filter with low-pass characteristics was implemented as cascaded second order systems. The Nyquist fraction value was chosen to be 0·03.

Both bending moments and accelerations have been measured simultaneously when the vehicle moves across the bridge at different speeds. After moving axle loads were identified from measured responses based on the four identification methods, respectively, the responses were then rebuilt from identified loads and the relative percentage errors (RPE) between the rebuilt and the measured responses were evaluated. In this comparative study, the results were only based on measured bending moments. The acceptable maximum RPE is within 10%. The results associated with accelerations will be reported separately.

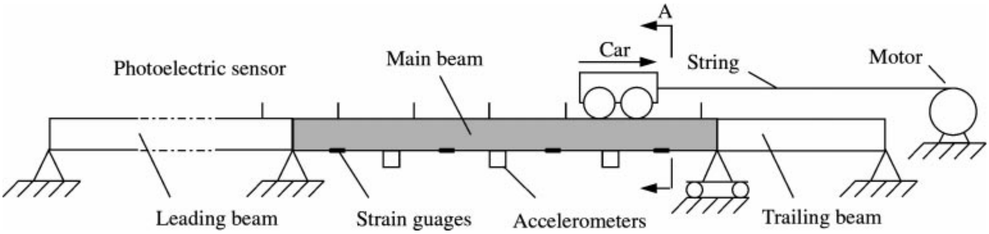


Figure 5. Experimental set-up for moving force identification.

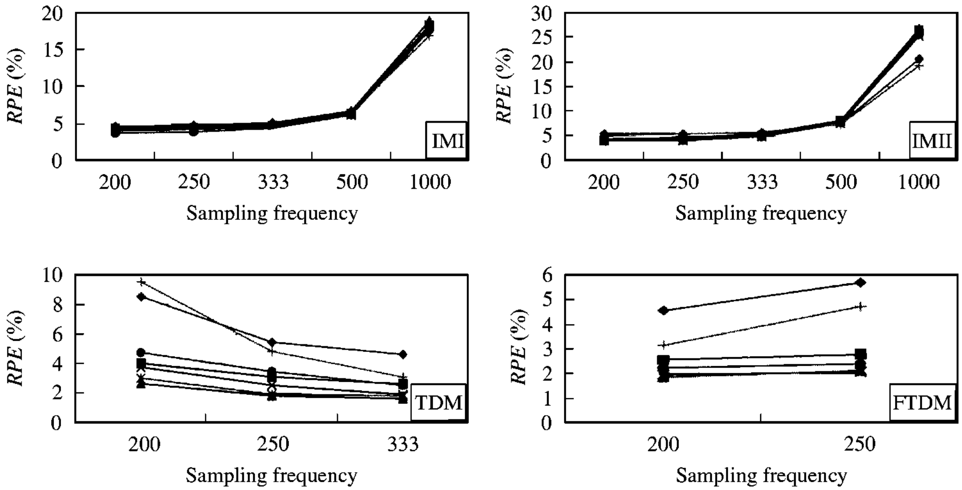


Figure 6. Effect of sampling frequency. —◆—, sta. 1; —■—, sta. 2; —▲—, sta. 3; —×—, sta. 4; —*—, sta. 5; —●—, sta 6; —+—, sta 7.

Since many parameters play an important role in the moving force identification, this comparative study is only to investigate effects of several main parameters on the four methods. These parameters include sampling frequency, mode number involved in identfication calculation, car speed, measuring sensor number and location. The parameters were studied one at a time.

5.1. EFFECT OF SAMPLING FREQUENCY

As there is a computer memory problem in calculating the inverse of a large matrix for TDM and FTDM, the maximum sampling frequency is limited to be within 500 Hz. The sampling frequencies of 200, 250 and 333 Hz were set here. There is no memory problem for IMI and IMII, the sampling frequencies of 200, 250, 333, 400, 500, and 1000 Hz were tested. Figure 6 illustrated in *RPE* values of the four identification methods with various sampling frequencies, where car speed is 10 Units (1 Unit $\cong$ 0·102 m/s) for both IMI and IMII, but 15 Units for both TDM and FTDM. The mode number is $MN = 3$ for IMII, but $MN = 5$ for both TDM and FTDM. Obviously, the effect of sampling frequency on both IMI and IMII is not significant within 300 Hz. It is independent of sampling frequency. But after that, it obviously increases with an increase in sampling frequency. Especially at 1000 Hz, the *RPE* values are not acceptable. A similar conclusion is eligible for FTDM but FTDM is not effective as sampling frequency increases up to 333 Hz. This shows that IMI, IMII and FTDM are suitable for a lower sampling frequency; the highest accuracy corresponds to the case of the lowest sampling frequency of $f_s = 200$ Hz. The effect of sampling frequency on TDM is completely different from the above three methods. The higher the sampling frequency, the lower are the *RPE* values for all the measuring stations. It shows that TDM is suitable for the higher sampling frequency.

5.2. EFFECT OF MODE NUMBER (*MN*)

The IMI is independent of the mode number, so it is not incorporated here. Generally, the mode number involved should be bigger than and equal to one more than axle number of

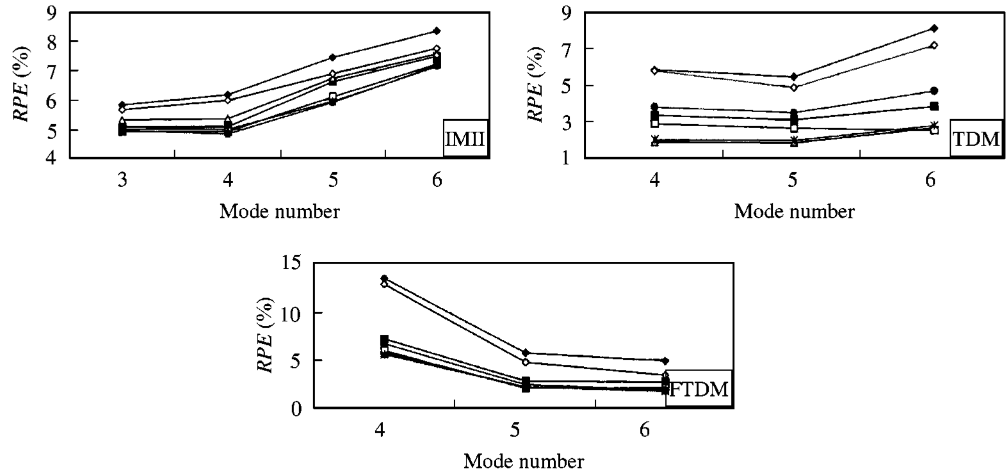


Figure 7. Effect of mode number: —◆—, sta. 1; —■—, sta. 2; —△—, sta. 3; —□—, sta. 4; —*—, sta. 5; —●—, sta. 6; —○—, sta. 7.

a vehicle. Figure 7 shows the effect of mode number on IMII, TDM and FTDM, in which *MN* is from 3 through 6. The speed is 10 Units and the sampling frequency is 333 Hz for IMII, values of 250 Hz and 15 Units were chosen for both TDM and FTDM. It can be seen that the *RPE* increases gradually with increase in mode number at all seven measurement stations for IMII. If the mode number is less than or equal to 3, the *RPE* values are not acceptable and both the TDM and FTDM failed to identify the two moving forces. However, if the mode number is larger than 3, such as 4, 5 and 6 as shown in Figure 7, the *RPE* values reduce dramatically. Further, the *RPE* values increase gradually with increase in the mode number for TDM while they decrease slightly with increase in the mode number for FTDM. It shows that these two methods are effective if the required mode number is achieved or exceeded but otherwise fail.

5.3. EFFECT OF CAR SPEED

When the test was carried out, three vehicle speeds were set manually at 5, 10 and 15 Units respectively. After acquiring the response data, the car speed was calculated and the uniformity of speed was checked. If the speed was stable, the experiment was repeated 5 times for each speed case to check whether or not the properties of both structure and measurement system had changed. If no significant change was found, the corresponding recorded data were accepted for the moving force identification. Figure 8 shows a summary of *RPE* values at some specific car speeds, where the sampling frequency is 250 Hz for both IMI and IMII, but 200 Hz for both TDM and FTDM. Mode number is 3 for IMII but 4 for both TDM and FTDM. Case "5-2" means the *second set* of data was recorded when the vehicle moves across the bridge at the speed of 5 Units. Others are similarly identified. It is interesting to note that the *RPE* values first decrease and then increase with increase in car speed for IMI. But the change in the *RPE* values is not so significant although decreasing slightly with increase in car speed for both IMII and TDM. It may be concluded that both IMII and TDM are independent of car speeds. However, this change is dramatic for FTDM. FTDM failed to identify the forces while the vehicle speed is lower, say 5 Units, but the identified results are getting better and better as the vehicle speed increases. Fortunately, the identified result is acceptable at last in the case of 15 Units for the FTDM.

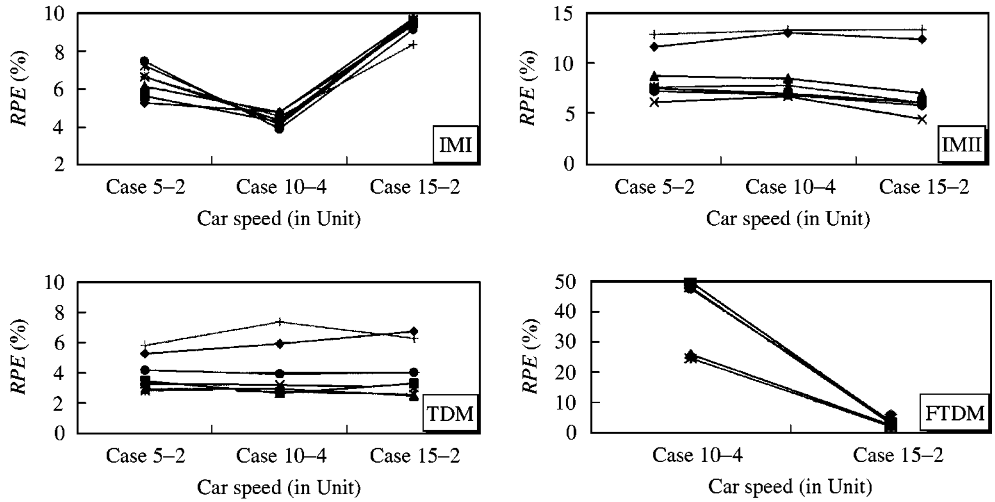


Figure 8. Effect of car speed: —◆—, sta. 1; —■—, sta. 2; —▲—, sta. 3; —×—, sta. 4; —*—, sta. 5; —●—, sta. 6; —+—, sta. 7.

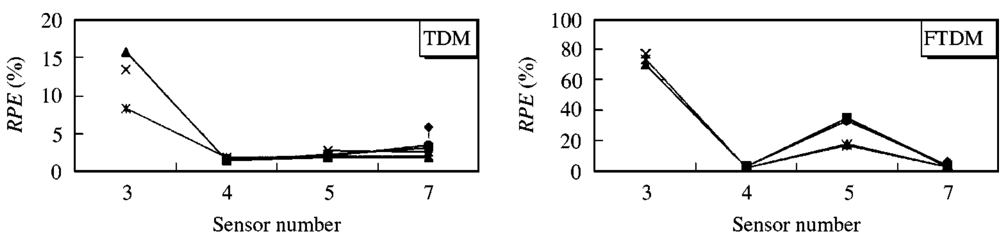


Figure 9. Effect of sensor number.

5.4. EFFECT OF VARIOUS SENSORS

The effects of sensors on IMI and IMII can be found in references [8, 14] respectively. For both TDM and FTDM, the *RPE* values are shown in Figure 9, in which the sensor number N_l was set to 2, 3, 4, 5, and 7 respectively while the other parameters $MN = 5$, $f_s = 250$ Hz, $c = 15$ Units were not changed for all study cases. It shows that TDM requires at least three, best have four measurement stations to obtain the two correct moving forces. FTDM should have at least one more measurement station than when using TDM, i.e., 4, to obtain the same number of moving forces. However, the *RPE* errors are increased obviously when measurement station number is equal to 5. This is because the addition of the fifth station is placed on the middle span $1/2L$, namely the node of second and fourth modes of supported beam. Nevertheless, when $N_l = 7$, i.e., put two more stations at $1/8L$ and $7/8L$, respectively, the *RPE* values by FTDM recover normal level to within 10%. It indicates that FTDM is sensitive to the locations of the measuring station and they should be selected carefully. In general, for TDM and FTDM, the identification accuracy is better if measured response data at more measuring stations are adopted.

6. CONCLUSIONS

A comparative study of the four moving force identification methods is complete. The illustrative examples and experiments in laboratory show that the accuracy of all methods is independent of the sampling frequency and both IMII and TDM are further independent of the speed of vehicles. The accuracy of IMI is significantly affected by the ASSR and noise. If the number of correctly identified bridge vibration modes is equal to the number of sensors IMII will be good for applications to any ASSR and a low level of noise, otherwise an error will be induced from two inversions of rectangular matrices. TDM is good for any case as concluded above, but it is a time-consuming method. The recommended number of sensors is the number of axles plus one for TDM and two for FTDM. The best combination of sensors is for all sensors to be accelerometers. Results show that IMI and IMII are less sensitive than TDM and FTDM to errors in speeds. Similarly, TDM is the most insensitive among IMI, IMII, and FTDM using an axle spacing with errors in force identification. For errors in the locations of sensors, TDM is less sensitive than FTDM, IMI and IMII.

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